ACOUSTIC CONDUCTANCE OF BURNING PROPELLANT SURFACE

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The occurrence of acoustic combustion instability in solid propellant rocket engines is associated with the possibility of amplification of sound waves upon reflection from the burning surface of the solid propellant. Several experimental methods have been proposed for measuring the acoustic conductivity of the burning surface of solid propellants, and values of this quantity have been presented in the literature (for example, [1-3]) for several solid propellants over a wide frequency range.

The problem of theoretical determination of the magnitude of the acoustic conductivity in the combustion zone reduces to study of the restructuring of the physicochemical processes in the combustion zone under the influence of the harmonic pressure disturbances and calculation of the magnitude of the acoustic perturbation of the gas efflux velocity from the combustion zone. Several studies ([4-7], for example) have been devoted to the calculation of the acoustic conductivity of the solid propellant burning surface. In these studies the acoustical conductivity is found using the solid propellant combustion models which contain several constants, for example such as the activation energies of the chemical reactions. By choice of these characteristics of the solid propellant combustion zone we can obtain satisfactory agreement between the experimental and theoretical data. However, such comparison of theory and experiment does not permit drawing any definite conclusion on the validity of the unsteady solid propellant combustion models used, so the question of the region of applicability of these models remains unresolved.

A model of unsteady solid propellant combustion based on the use of the steady-state dependences of the combustion velocity and solid propellant surface temperature on the pressure and initial temperature was proposed in [8, 9]. These relationships were determined experimentally for ballistite propellant in [10]. This makes it possible in the case of ballistite propellant to obtain an analytic formula and numerical values of the acoustic conductivity on the basis of quite definite experimental values of all the parameters which appear in the governing equations. In the following we present the results of such a calculation, obtained using the basic assumptions and equations of [6].

1. Equations. The system of equations for determining the acoustic conductivity of the burning surface of a solid propellant consists of five equations for the burning surface temperature perturbation δT_s, the solid propellant burning velocity perturbation δU, the temperature gradient perturbation $\delta\varphi$ at the burning surface inside the condensed phase, the perturbation δu of the gas efflux velocity from the burning surface, and the pressure perturbation δp.

Three of these equations express very general properties of the solid propellant burning zone and are independent of the concrete assumptions on the connection between the combustion velocity and surface temperature and the conditions in the burning zone. These are the equations describing the restructuring of the thermal layer in the condensed phase, the thermal energy balance in the inertialess part of the burning zone, and the mass balance on the burning surface. In accordance with the results of [6], we write these equations in the form

$$\frac{\delta\varphi}{\varphi} - \frac{\beta_1}{2}\frac{\delta T_s}{T_s - T_0} + \frac{i(\beta_1 - 2)}{2\Omega}\frac{\delta U}{U} = 0 \quad \left(\beta_1 = 1 + \sqrt{1 + 4i\Omega},\ \Omega = \frac{\varkappa\omega}{U^2}\right) \tag{1.1}$$

$$\frac{\delta\varphi}{\varphi} - \frac{\delta T_s}{T_s - T_0} - \frac{\delta U}{U} + \frac{1}{\tau}\frac{\delta T_2}{T_2} = 0 \quad \left(\varphi = \frac{U}{\varkappa}(T_s - T_0)\right) \tag{1.2}$$

$$\frac{\delta u}{u} - \frac{\delta U}{U} + \frac{\delta p}{p} - \frac{\delta T_2}{T_2} = 0 \quad \left(\tau = \frac{c_1(T_s - T_0)}{c_2 T_2}\right) \tag{1.3}$$

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TABLE 1

	μ	k	τ	r	ν	T_0, °C	p, atm	U, mm/sec
1	0.013	2.00	0.30	0.16	0.67	—100	1	0.28
2	0.023	1.88	0.24	0.24	0.67	0	1	0.60
3	0.028	2.24	0.21	0.51	0.67	50	1	1.0
4	0.17	0.57	0.23	0.26	0.67	—100	20	19
5	0.21	1.04	0.20	0.36	0.67	—50	20	21.5
6	0.32	1.38	0.18	0.47	0.67	0	20	26
7	0.36	1.99	0.17	0.60	0.67	50	20	35
8	0.32	2.34	0.15	0.70	0.67	100	20	49

TABLE 2. Values of 10^3 R

Hz	0	5	10	20	40	60	80	100
1	0.12	1.13	0.40	0.11	0.06	0.07	0.08	0.09
2	0.18	0.98	0.04	0.02	0.07	0.11	0.13	0.15
3	0.31	—8.10	—0.51	—0.09	0.07	0.14	0.18	0.20
4	0.39	0.19	0.10	0.11	0.22	0.32	0.40	0.45
5	0.46	0.17	—0.07	—0.14	0.21	0.48	0.64	0.75
6	0.56	0.30	—0.03	—0.27	0.53	1.14	1.44	1.61
7	0.75	0.60	0.27	0.70	—1.79	1.68	3.16	3.54
8	1.08	1.02	0.86	0.33	—1.67	—5.40	—4.06	1.87

Here T_S, T_0, T_2, U, u, p are the steady-state values of the temperature at the surface of the solid propellant, initial temperature, burning temperature, burning velocity, gas efflux velocity, and pressure, respectively; c_1 is the specific heat of the condensed phase; c_2 is the specific heat of the gas; $\varkappa$ is the thermal diffusivity of the condensed phase, ω is the oscillation frequency. We note that (1.3) is written on the assumption that waves of two types – acoustic and entropy – propagate in the combustion products. The necessity for accounting for the entropy waves was pointed out in [11]. The existence of the entropy waves was recently established experimentally.

In deriving the other two equations for the perturbations δT_S, δU, $\delta\varphi$, and δp we shall use the basic assumption of the theory developed in [8, 9]. In [8, 9], along with several assumptions coinciding with those adopted in [6], it is postulated that the steady-state dependences $U(p, \varphi)$ and $T_S(p, \varphi)$ of the burning velocity and burning surface temperature on the pressure and temperature gradient at the solid propellant surface remain valid under unsteady conditions as well. In this case the form of the relations $U(p, \varphi)$ and $T_S(p, \varphi)$ is determined by excluding the initial temperature T_0 from the dependences $U(p, T_0)$ and $T_S(p, T_0)$ of the burning velocity and surface temperature on the pressure and initial temperature with the aid of the relation

$$\varphi = (T_s - T_0)\, U / \varkappa$$

Using this hypothesis, we can obtain the equations

$$\frac{\delta T_s}{T_s - T_0} - \frac{\mu(k-1) - \nu r}{k + r - 1}\frac{\delta p}{p} - \frac{r}{r + k - 1}\frac{\delta\varphi}{\varphi} = 0 \quad (1.4)$$

$$\frac{\delta U}{U} - \frac{\nu(r-1) - \mu k}{k + r - 1}\frac{\delta p}{p} - \frac{k}{k + r - 1}\frac{\delta\varphi}{\varphi} = 0 \quad (1.5)$$

Here

$$k = (T_s - T_0)\left(\frac{\partial \ln U}{\partial T_0}\right)_p, \quad \nu = \left(\frac{\partial \ln U}{\partial \ln p}\right)_{T_0}, \quad r = \left(\frac{\partial T_s}{\partial T_0}\right)_p, \quad \mu = \frac{1}{T_s - T_0}\left(\frac{\partial T_s}{\partial \ln p}\right)_{T_0}$$

The parameters k, ν, r, μ are found from the known steady-state relations $U(T_0, p)$ and $T_S(T_0, p)$. Equations (1.1)-(1.5) make it possible to calculate the acoustic conductivity of the solid propellant burning surface.

2. Acoustic Conductivity. By definition the acoustic conductivity of a surface, written in dimensionless form, is

$$\zeta = -\rho c \frac{\delta u}{\delta p} \quad (2.1)$$

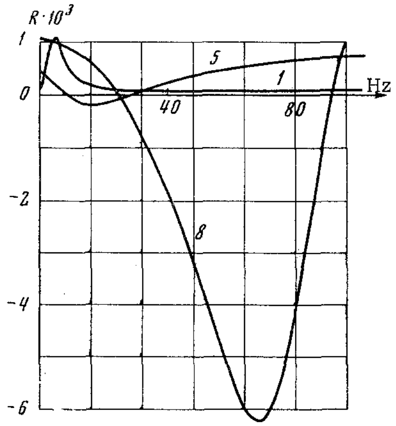

Fig. 1

Here δu, δp are the acoustic perturbations of the velocity and pressure at the surface in question, ρ is the density of the gas, c is the speed of sound. Usually the acoustic conductivity has a complex value. Upon reflection from the surface the sound wave is amplified if the real part $Re\,\xi$ of the acoustic conductivity of this surface has a negative value $Re\,\xi < 0$. Therefore the tendency of a solid propellant toward acoustic combustion instability is usually characterized by the magnitude of the real part of the acoustic conductivity. It is not difficult to obtain from (1.1)-(1.5), with account for the definition (2.1), the explicit expression for the acoustic conductivity and then, after separting the real part, obtain the formula

$$Re\left(\frac{c}{\gamma u}\zeta\right) = R = \frac{a_1x^4 + b_1x^3 + c_1x^2 + d_1x + l_1}{a_2x^4 + b_2x^3 + c_2x^2 + d_2x + l_2}\left(\gamma = \frac{c_p}{c_v}\right) \quad (2.2)$$

$$x = [1/2 + 1/2\ (1 + 16\Omega^2)^{1/2}]^{1/2}, \qquad a_1 = (\mu\tau + r + \mu k - r\nu)\, r / 2$$

$$b_1 = (\mu\tau + \mu k - \mu k\tau - 2kr - \nu\tau r - \mu k^2 + 2r - 2r\nu + \nu kr) / 2$$

$$c_1 = (k - 1)\ (\nu + \nu\tau - 1 + k) - a_1, \qquad d_1 = 1 - k^2 - b_1 - \nu - \nu k\tau, \quad l_1 = \nu\tau + 2k - k\nu,$$

$$a_2 = r^2 / 2, \qquad b_2 = r\,(1 - k), \quad c_2 = (1 - k)^2 - r^2 / 2, \qquad d_2 = 1 - k^2 + rk - r, \qquad l_2 = 2k$$

3. Computation Results and Discussion. Formula (2.2) was used to calculate the magnitude of the real part of the acoustic conductivity of ballistite powder for eight different steady state burning regimes. The values of the parameters k, r, μ, τ, and also the values of the quantities T_0, p, U for each regime, determined from the experimental data of [10], are presented in Table 1.

Table 2 shows the values of $10^3 \cdot R$ in the frequency range from 0 to 100 Hz for each of the eight regimes in question.

Figure 1 shows the real part of the acoustic conductivity as a function of frequency for regimes 1, 5, 8. The calculation was made over a frequency range considerably broader (up to 10^4 Hz) than the frequency range shown in Table 2 and in the figure. Data for frequencies higher than 100 Hz are not presented, since in this region the calculation always led to positive values of the magnitude of the real part of the acoustic conductivity. A marked change of the value of R takes place only in the frequency range up to the order of 100 Hz.

With further increase of the frequency the quantity $10^3 \cdot R$ approaches a constant value which depends on the combustion regime. For example, for regimes 1, 5, 8 the corresponding values of $10^3 \cdot R$ are 0.18, 1.41, and 4.54. Analysis of these results shows that the tendency of the system to amplify acoustic oscillations increases with increase of μ, k, r, and reduction of τ. Moreover, the pressure has a marked influence on the acoustic instability of the system. Of the five combustion regimes with pressure p = 20 atm four have an instability region in the range from 10 to 80 Hz. For three combustion regimes with pressure p = 1 atm amplification of the acoustic oscillations was detected only in one case.

It follows from the calculation results that the formula for the acoustic conductivity obtained using the unsteady combustion model of [8, 9] and the experimental data of [10] leads either to positive values of the real part of the acoustic conductivity of the burning surface or indicates the possibility of amplification in the low-frequency region (up to 100 Hz). The experimental data indicate acoustic combustion instability over a broader frequency range. More detailed information on the parameters μ, k, r, τ, ν are required for a final comparison of theory and experiment, since the information which can be obtained from the experimental data of [10] are not sufficiently complete.

We note also that the discrepancy between theory and experiment for unsteady processes with characteristic time shorter than 10^{-2} sec can apparently be associated with the incompleteness of the combustion model used as a basis for the calculation, in which all the processes in the combustion reaction zone, including the chemical reactions in the condensed phase, are considered quasistationary.

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